

Computing with Coupled Oscillators: Theory, Devices, and Applications

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Abstract—This paper will give a review of recent work on using networks of coupled oscillators for analog information processing. We will discuss the rationale of using coupled oscillators, and how they can be used to perform computational tasks, such as associative computing primitives, or how they can serve as hardware accelerators in vision processing pipelines. Further, we will study two specific physical implementations for such oscillator, namely relaxation oscillators based on metal-insulator phase transitions and magnetic spin-torque oscillators. We will also discuss the potential of such coupled-oscillator networks to solve computationally-hard optimization problems or even NP-hard problems.

Keywords: *coupled oscillator networks, non-Boolean computing, spin-torque oscillators, relaxation oscillators*

I. INTRODUCTION

Traditionally, the search for beyond Moore devices sought a potential replacement for MOS transistors or logic gates, which used emerging and often non-electrical state variables. As there is still no clear winner among the proposed emerging devices, more radical ideas are being explored, which do away with the concept of level-based, Boolean computing and even with the principle that a device with a nonlinear current-voltage characteristics should be at the main building block of a computing architecture. One of these radically new computing technologies is oscillator-based computing – where the building blocks are simple, compact nanoscale oscillators (many of them) and the computation itself is done by the nonlinear interaction of these oscillatory building blocks [1].

There are multiple motivations for oscillatory computing. Firstly, oscillators are ubiquitous in the physical world and in fact a large number of phenomena is understood in terms of interacting oscillator dynamics. Secondly biological systems (including the human brain) seem to communicate with each other via oscillatory signals. This suggests that phase and frequency-based encoding of information may have inherent benefits. The third argument is that spike-based signal processors were among the most successful implementations of analog signal processors – and oscillator based computing furthers this direction.

In this paper, we would like to show two representative examples of oscillatory computing. The two approaches are,

in many respects, orthogonal to each other. The first approach targets hardware accelerators for vision processing pipelines – this should be a relatively simple network of fast, low-power spin-based oscillators and in terms of performance / power ratio it should significantly outperform any transistor-based solution. The second approach targets computationally hard (possibly NP hard) problems and relies on the energy minimization property of the network via nearest neighbor interaction. This latter computing scheme was demonstrated using relaxation oscillators, built from hysteretic switches relying on metal-insulator phase transition.

The oscillator-based computing models we will outline below represent one possible approach to neuromorphic (or neurally inspired) computing. But rather than trying hard to reproduce behaviors observed in biological systems, we put a strong focus on realizing these circuits using emerging oscillatory devices as building blocks. The combination of emerging hardware with such non-conventional models of computation, may hold the key for ultra-low-power, special purpose computing devices.

II. BUILDING BLOCKS OF AN OSCILLATORY COMPUTER

Since a practical computing device may contain millions or possibly billions of oscillatory building blocks, the oscillatory building blocks have to meet high requirements: they should be low-power, high-frequency, programmable and with read-out capability in an energy-efficient way. It is hard to find a physical oscillator satisfying all these requirements simultaneously.

A) Spin-based oscillators

Spin-based oscillators use the magnetic degree of freedom for doing computation [2]. Spin torque oscillators (STOs) use spin-polarized current to generate precession of magnetic moments in a magnetic thin-film in a sandwich structure, such as the one shown in the inset of Fig. 1. Upon precession of the magnetic moment, the resistance of the sandwich is changing and this allows electrical pickup of the magnetization signal. As the current through the oscillator changes, so does the precession frequency of the oscillator, i.e. it acts like a voltage controlled oscillator, as shown in Fig 1. The figure also shows an example of controlling the oscillator by an externally injected AC signal – a small-amplitude signal with $f=15$ GHz frequency, which is superposed to the DC driving signal of the

oscillator, locks the frequency of the STO for a range of driving currents.

It is possible to enable these oscillators to interact with each other via electrical coupling. One possible way to do that is to use the magnetic field of a nearby electrical wire – the wire itself is fed by the signals picked from the oscillators. A sinusoidal current running through these wire may lock the oscillators frequency and provides a mean to control or to interconnect oscillators [4].

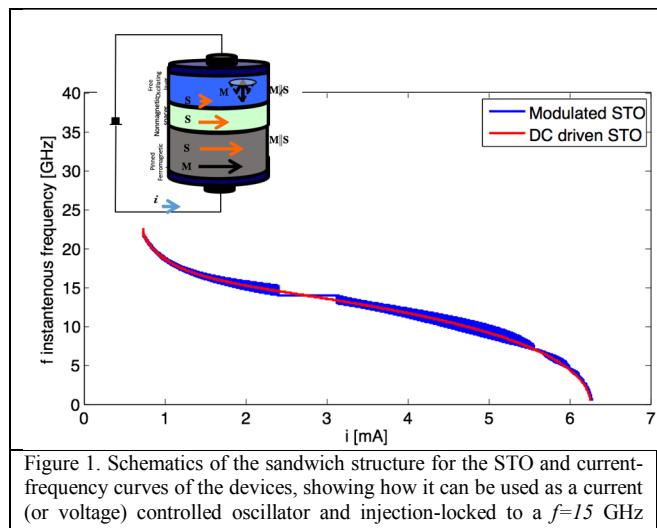


Figure 1. Schematics of the sandwich structure for the STO and current-frequency curves of the devices, showing how it can be used as a current (or voltage) controlled oscillator and injection-locked to a $f=15$ GHz source.

There are many variants of spin-based oscillators, using different mechanisms to power the magnetization oscillations - such as spin-orbit torque instead of spin-torque. These latter ones have different geometry and may have somewhat better power figures. Also, different magnetization configurations may be used (macrospin and vortex oscillations) and one may use different electrical or direct magnetic interconnections between the devices. Spin-torque oscillators (especially the ones based on macrospin oscillations) are inherently high-frequency devices (frequency in the 10 GHz range) and generate relatively weak (millivolt-range) signals. Engineering the electrical coupling and read-out network for those oscillators is a challenging task [3].

B) Relaxation oscillators from phase-change materials

Relaxation oscillators may be made from transistor-based circuitry, but a much more compact and scalable is to use hysteretic (memristive) switches as the active element in the oscillator. Such switches can be realized from phase change materials and they give a highly nonlinear switching characteristics. The device construction is rather simple: the switching layer have to be sandwiched between metallic electrodes – the simple device structure may result in ultimately high scalability. An example of the electrical switching behavior is given in which is illustrated in Fig 2a).

The switch changes between a low and high resistance state for well-defined voltage thresholds.

To turn the device into a relaxation oscillator, one may use the switch in the configuration of Fig. 2b. Maffezoni et. al. [5] gives a detailed analysis of the parameter ranges when such system produces oscillations – we use his parameters in the analysis below. For the appropriate circuit values, the RC element is periodically charged and discharged producing an oscillatory waveform. Those are shown in Fig 2c, also for a situation where the parameters are taken from Maffezoni [5].

Just as in the case of the STOs, the voltage of the power supply can be used to tune the oscillation frequency and phase-change oscillators function as nanoscale voltage controlled oscillators. AC current injected in the oscillator (i.e. an AC voltage source that is superposed to the driving source) will lead to injection locking.

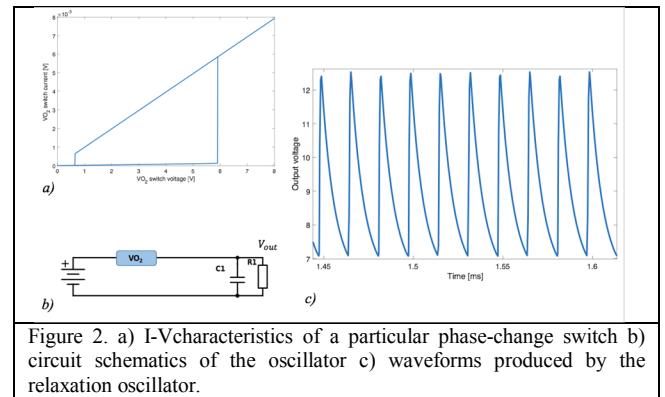


Figure 2. a) I-Vcharacteristics of a particular phase-change switch b) circuit schematics of the oscillator c) waveforms produced by the relaxation oscillator.

III. INTERCONNECTION OF OSCILLATOR NETWORKS

A) Spin-torque oscillator interactions

Interconnections between devices that use non-electrical state variables, is more challenging as it either requires converting the signal to the electrical domain, transmitting it and then converting it back to control the oscillators – which may result in significant overhead in the circuitry.

One example of an interacting STO system is given in Fig 3. Three independently biased STOs are shown, their oscillatory signal is picked up by an RC filter and the signals are superposed at a summing node (see Fig 3b). Due to the relatively inefficient magnetoelectric interconversion, the amplitude of the oscillating voltage is going to be typically only a few percent of the bias voltage and an amplifier is required in the feedback signal path.

In order to minimize the amount of on-chip microwave circuitry, the feedback is done by magnetic field-coupling to an electrical wire. The geometry is indicated by the sketch of Fig 3a. The magnetic field of the wire can lock nearby STOs without the need if individual contact to each of them [4] [6].

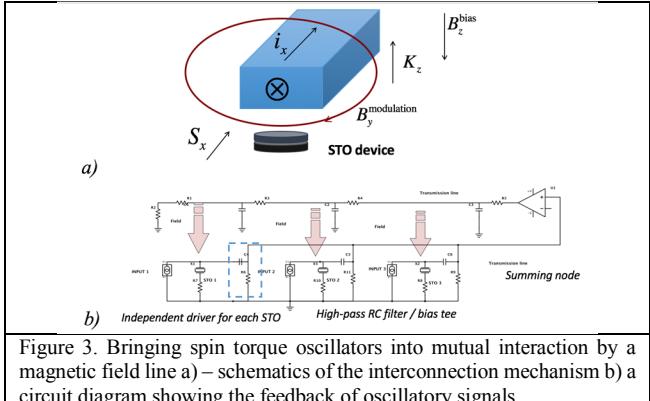


Figure 3. Bringing spin torque oscillators into mutual interaction by a magnetic field line a) – schematics of the interconnection mechanism b) a circuit diagram showing the feedback of oscillatory signals.

B) Interconnection of relaxation oscillators

Electrical oscillators are more straightforward to interconnect and in the case of the studied relaxation oscillators, no amplification is needed as the passive interconnections are sufficiently strong to bring them into interaction.

Depending on the phase shift introduced by the interconnections, one can achieve in-phase or out of phase synchronization between the oscillators.

As an example, Fig 4 shows how capacitive interconnections between the oscillators drive them toward an anti-phase synchronization. The capacitive coupling also introduces an additional time constant in the circuit, changing the waveform. This foreshadows future scalability issues – a large number of interconnections may lead to overly complex dynamics and possible quenching of the oscillations rather than just bringing oscillators into in or out of phase oscillations.

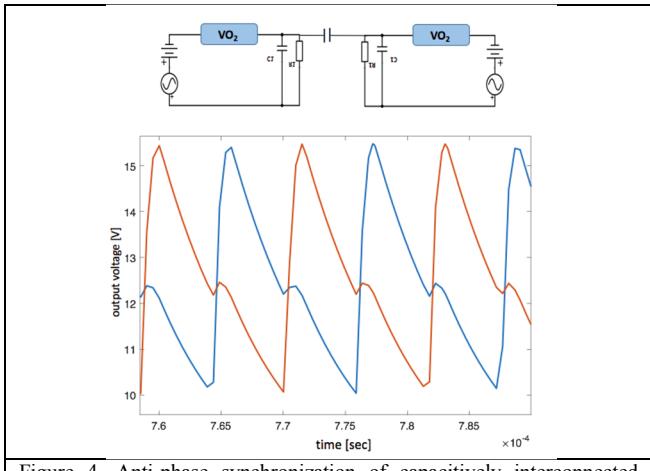


Figure 4. Anti-phase synchronization of capacitively interconnected oscillators. The capacitor also alters the oscillator waveforms.

Resistive interconnections lead to in-phase synchronization and for weak couplings, unlike capacitive ones, they preserve the original waveform of the oscillators.

IV. COMPUTING ARCHITECTURES FOR OSCILLATORS

We examine two different computational models that are based on coupled oscillators. In the first scheme interaction

nonlinear interaction of oscillators serves an analog way to determine the distance (degree of match) for the output – potentially in a lot more efficient way than by a number crunching digital computer. In the second scheme, oscillator interactions solves an optimization problem by the nonlinear dynamics – such optimization problems, when scaled up, are exponentially difficult for digital circuitry.

A) Hardware accelerators for calculating a distance metric

A common task in image processing is the calculation of the Euclidean distance between two vectors. While this can be straightforwardly done by digital circuitry, it is computationally expensive. This is especially problematic at the front end of an image processing pipeline, where a large amount data (hi-resolution images at high frame per seconds) has to be handled – and these simple, repetitive tasks may account for the vast majority of the consumed power .

Euclidean distance calculation can be done by the STO circuit of Fig 3b. Elements of the two vectors are represented by analog current vectors ($\mathbf{i}_A = (i_{A1}, i_{A2}, i_{A3}, \dots)$ and $\mathbf{i}_B = (i_{B1}, i_{B2}, i_{B3}, \dots)$) and the element-wise difference of the two vectors is applied as input to the STOs, in addition to a static bias current. The amplifier output of Fig 3b is connected to a diode-based power detector and its output signal is the of the degree of matching.

If the \mathbf{i}_A and \mathbf{i}_B vectors are similar to each other (i.e having a small distance) then the nominally identical STOs experience only nominally identical bias currents and easily frequency and phase-lock to each other, forming a coherent oscillatory state. The output of the power detector measures the degree of coherence between the oscillators: if all STOs are synced, then the output is maximal, and a growing distance between the current vectors will result in less perfect synchronization between the STOs and a smaller output signal. The exact shape of the curve depends on the STO characteristics, the coupling strength to the field line and the amplifier gain and phase shift. As an example, Fig. 5 shows the output at the power detector as a function of the absolute value of the distance between the current vectors and for different amplifier delays. The parabolic shape of the curve (for small distances) indicates that it is in fact a good approximation to Euclidean distance.

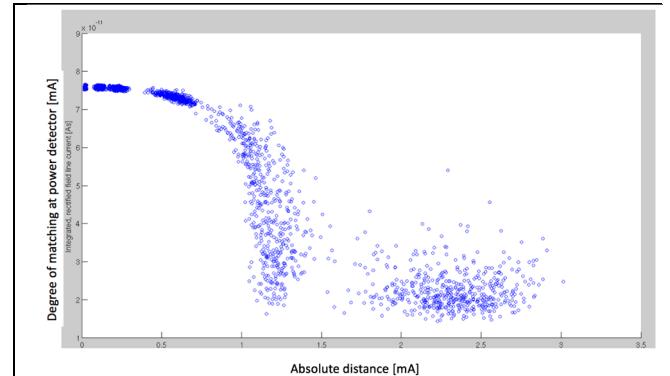


Figure 5. Output of an STO array, which characterizes the degree of synchronization between the oscillators and calculates the degree of matching in an analog way.

Dot products can be calculated straightforwardly from Euclidean distances – and dot product calculation is one of the most-needed operation for convolutional neural networks and deep-learning nets. Development of a special-purpose circuit for this operation seems well justified.

B) Computationally hard optimization problems

Combinatorial optimizations represent a problem class where an optimal value of a function, or its optimal point, needs to be computed within a domain set which is discrete or combinatorial. Vertex coloring of graphs is a combinatorial optimization problem which is NP-hard (non-deterministic polynomial-time hard). Vertex coloring is also one of the most studied NP-hard combinatorial optimization problems not only for its significance in computational theory but also for its many real world applications like scheduling and resource allocation. For any deterministic system to be able to solve such hard problems, exponential resources are required which can be in terms of time, hardware components, maximum magnitudes of variables or their precision. Recently, we have used an array of coupled relaxation oscillators fabricated using Vanadium dioxide (VO_2) metal-insulator-transition devices and coupled capacitively, can lead to system dynamics on which vertex coloring of unweighted and undirected graphs (hitherto referred to as the graph coloring problem) can be successfully mapped (Fig. 6). A VO_2 oscillator consists of a VO_2 device connected in series with a conductance g_s (with a loading capacitor C_L in parallel) and the output node is the node between the VO_2 device and the conductance (Fig. 6). Such a simple series circuit shows self-sustained relaxation oscillations.

We have recently demonstrated experimentally that when such relaxation oscillators are coupled using only capacitances in a manner topologically equivalent to an input graph, their steady state phases can be used to approximate the solution of the NP-hard minimum graph coloring problem. For this, we have to reformulate the graph coloring problem where instead of finding a color assignment for each node, we strive to find a circular ordering or circular permutation of the nodes such that the same colored nodes appear together in the ordering. Such a reformulation preserves the hardness of the problem and is useful for interpreting the output of our circuit. We have observed that the dynamics of such a coupled relaxation oscillator system is intrinsically connected to spectral algorithms for graph coloring, which use eigenvectors of adjacency matrix of the input graph to approximate the solutions.

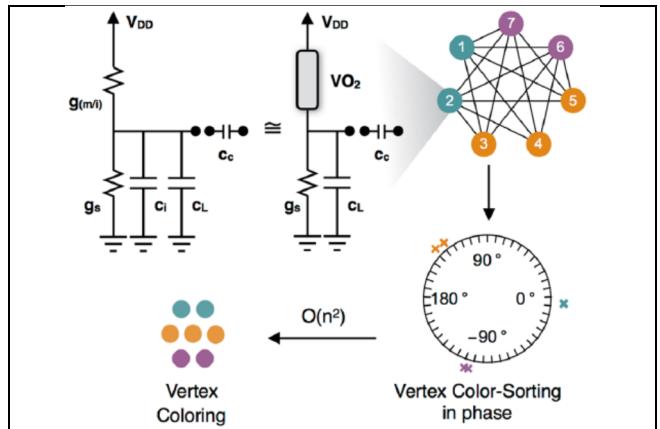


Figure 6. Overview of the system for vertex coloring and a simulation example. Relaxation oscillator are constructed from a series combination of VO_2 device and a resistor (with a loading capacitor in parallel), and are connected in a graph using capacitors. The equivalent circuit diagram of the VO_2 oscillator is shown using an internal capacitance c_i and a phase changing conductance $g_{(m)i}$ which switches between metallic conductance g_m and insulating conductance g_i .

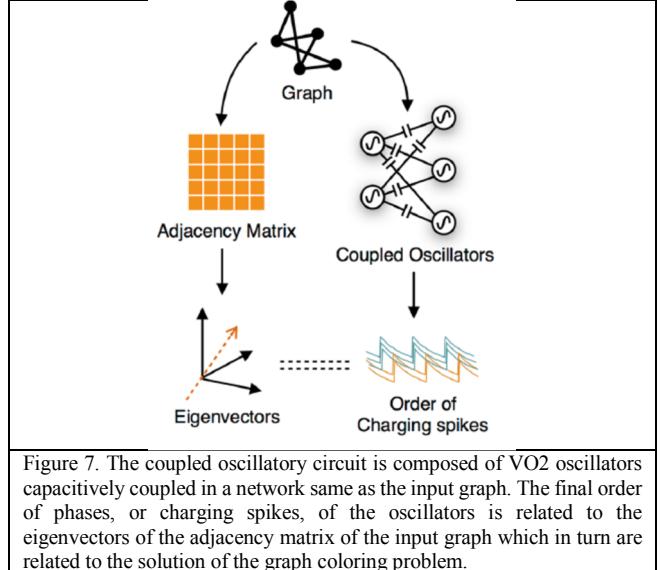


Figure 7. The coupled oscillatory circuit is composed of VO_2 oscillators capacitively coupled in a network same as the input graph. The final order of phases, or charging spikes, of the oscillators is related to the eigenvectors of the adjacency matrix of the input graph which in turn are related to the solution of the graph coloring problem.

Alternatively, the permutation of steady state phases of coupled relaxation oscillators depends on eigenvectors of the adjacency matrix in the same way as have been used by spectral algorithms for graph coloring (Fig. 7) [9]. There are a number of works in recent times now exploiting nonlinear dynamics for the solution of computationally hard (possibly NP hard) problems [10] Error! Reference source not found..

V. CONCLUSIONS – THE NEED FOR BENCHMARKING

One may rightfully ask about the benefits of oscillatory computing in general and about what makes the above-described architectures superior to digital or analog transistor-based circuitry.

Spin torque oscillator network are attractive, because the STOs operate at microwatt power and in the 10 GHz frequency range and at a footprint that is unachievable with CMOS based circuitry. Taking into account, however, the supporting circuitry (amplifiers, current generators that are

needed to control the network) significantly lessens the attractiveness of the circuit.

If the oscillator network solves a much harder problem (like the graph coloring discussed above) then the overhead associated with the CMOS supporting circuitry is less critical. It is important, however that the network is scalable well beyond toy problems – which remains a challenge, due to the complexity of the interconnect network.

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