Computing with Coupled Dynamical Systems

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I. Introduction

While Boolean logic has been the backbone of information processing, there are computationally hard problems like optimization and associative computing wherein this conventional paradigm is fundamentally inadequate. This results in computational inefficacy, and motivates us to explore new pathways to their solution. In this talk, we introduce an experimental testbed comprising of compact coupled relaxation oscillator based dynamical system that exploits the insulator-metal transition in the correlated material, vanadium dioxide (VO_2), to efficiently solve the approximate match between stored and input patterns. Our work is inspired by the understanding that associative computing finds a natural analogue in the energy minimization processes of parallel, coupled dynamical systems. Our work not only elucidates a physics-based computing method but also presents opportunities for building customized analog co-processors for solving computationally hard problems efficiently.

II. Low Power VO_2 relaxation oscillator

VO_2 exhibits an electrically induced insulator-metal transition (IMT) characterized by a large and abrupt change in resistivity (Fig. 1(a)(b)). Further, connecting a resistive load like a MOSFET in series with the VO_2 induces a negative differential resistance (NDR) across the phase transition which can enable sustained relaxation-type oscillations (Fig. 1(c)) [1]. Fig. 2 shows that low-power oscillators can be realized through dimensional scaling making VO_2 oscillators an attractive candidate for oscillator-based computing.

III. Associative Computing using Coupled Oscillators

We harness the phase synchronization dynamics of pairwise, capacitively coupled VO_2 relaxation oscillators (with MOSFET as the series resistive load) (Fig. 3(a)) to calculate a distance norm which is required for finding a 'degree of match' relevant to associative computing applications; the time domain waveforms of the synchronized oscillators are shown in Fig. 3(b) [2]. To understand the phase synchronization dynamics, we explore the phase space behavior of the coupled oscillators. The gray regions in the phase space indicate the XOR=0 regions. Figure 4(a) reveals that the steady-state periodic orbit of the coupled oscillators is highly sensitive to the gate input voltage difference ($=V_{GS,1}-V_{GS,2}$). When the input voltage difference is small ($V_{GS,1}=V_{GS,2}=0.3V$; $\Delta V_{GS}=0V$), the oscillators have a large out-of-phase locking character with only a small fraction of the periodic orbit lying in the gray region; as input voltages become dissimilar and the difference ΔV_{GS} increases, the periodic orbit evolves to having a higher percentage of in-phase locking character i.e. a larger fraction of the steady state orbit lies in the gray regions. To quantify the evolving periodic orbit, we define a time-averaged XOR metric which is calculated using the following steps: (1) threshold the analog output to a binary stream, (2) apply XOR operation on these binary values at every time instance (3) average the XOR output over a finite time duration. Figure 4(b) shows the experimental and simulated time-averaged XOR output of the coupled oscillators. Further, the qualitative match between the 2D contour map of the time-averaged XOR output of the VO_2 coupled oscillators with the distance function $(\sqrt{x_i}-\sqrt{y_i})^2$ indicates that oscillators compute a distance norm close to 0.5 i.e. $L_{0.5}$ (Fig. 5). The ability of oscillators to compute a distance norm can be used to find a degree-of-match which can be harnessed for associative processing applications like Visual Saliency. Figure 6 shows a representative image and its saliency map obtained using the dynamics of the coupled oscillator system.

IV. Graph Coloring using Coupled Relaxation Oscillators

We also investigate the phase synchronization properties of a larger system (more than two) of coupled VO_2 oscillators, and harness their dynamics to solve the vertex coloring of graphs, a prototypical combinatorial optimization problem. A graph is mapped onto the coupled oscillator hardware such that the oscillator represents the node, and the capacitive coupling represents the edge connecting the nodes in the graph (oscillator≡ vertex; capacitor ≡ edge). The resultant phase dynamics (distinct phase ≡ distinct color) of the synchronized oscillators then represent the coloring solution of the graph (Fig. 7). Figure 8 shows the coloring of a representative graph with three nodes. It can be observed that all the three oscillators that represent the graph have distinct phases implying that three colors are required to color the graph. Figure 9 shows the experimental coloring solution for various other graphs investigated in this work using the coupled oscillators.

V. Conclusion

In summary, we demonstrate compact and low-power VO_2 based coupled relaxation oscillators. Further, we experimentally investigate the phase synchronization dynamics of the coupled oscillator system (pairwise as well as larger systems of coupled oscillators) and show their application in solving associative computing problems like visual saliency, and combinatorial optimization problems like graph coloring.

Acknowledgement

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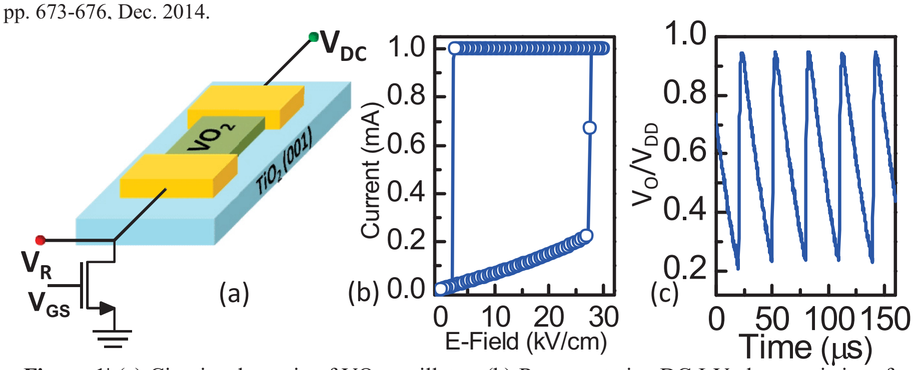

Figure 1| (a) Circuit schematic of VO_2 oscillator. (b) Representative DC I-V characteristics of VO_2 showing the insulator-metal transition. (c) Representative time domain waveform of the VO_2 oscillator.

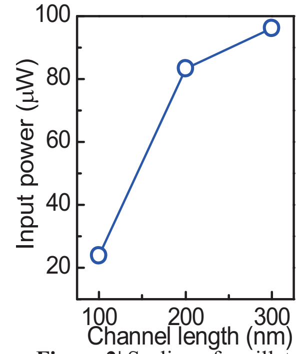

Figure 2| Scaling of oscillator input power with channel length of the VO_2 device.

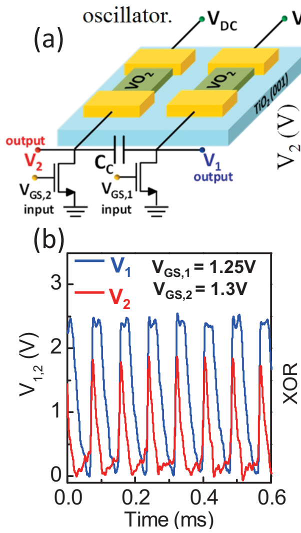

Figure 3| (a) Schematic of the capacitively coupled VO_2 oscillators. (b) Time domain waveforms of the synchronized oscillators.

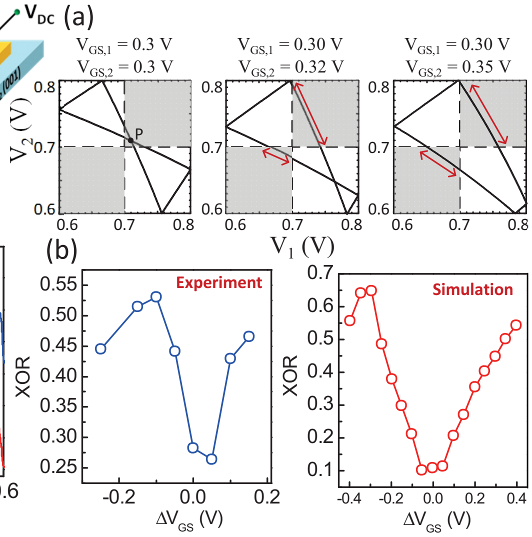

Figure 4| (a) Evolution of the steady-state periodic orbit of the coupled oscillators as a function of input voltage (difference) $V_{GS,1}$ and $V_{GS,2}$. (b) Experimental and simulated time-averaged XOR as a function of ΔV_{GS}.

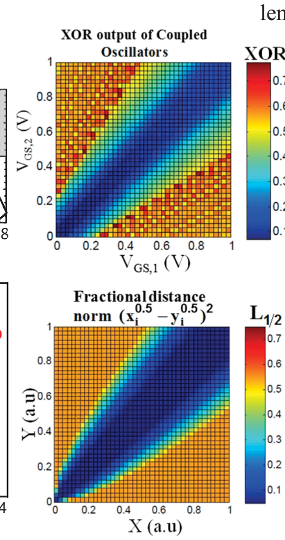

Figure 5| Fractional distance norm ($L_{0.5}$) calculated by the VO_2 coupled VO_2 oscillators.

Figure 6| Visual Saliency processing using VO_2 coupled oscillator dynamics.

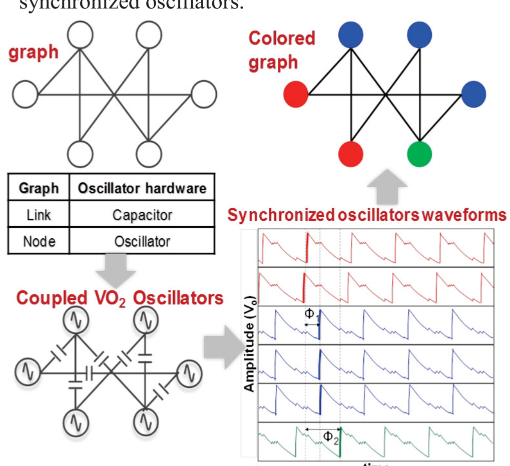

Figure 7| Vertex coloring of graphs using phase synchronization dynamics of VO_2 oscillators. The vertex of the graph is mapped to the oscillator; the edge is mapped to the coupling capacitor, and the phases of the oscillators reveals the coloring solution.

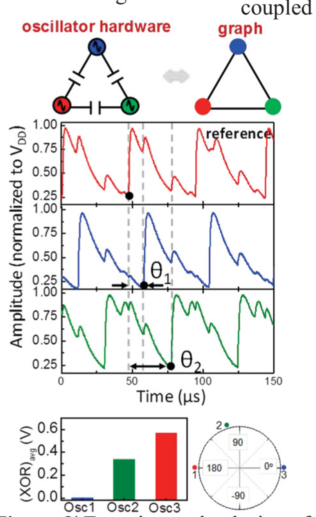

Figure 8| Experimental coloring of a graph with three nodes. The distinct phases represent distinct colors. Phase is measured using time-averaged XOR metric.

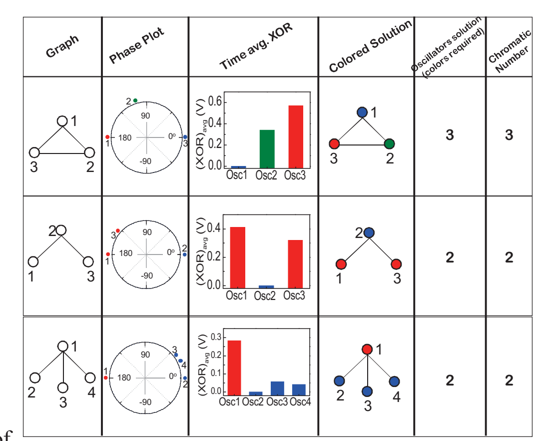

Figure 9| Experimental demonstration of the coloring of various graphs. Small phase differences for the same color can be attributed to variations.

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