Modelling hysteresis in vanadium dioxide oscillators

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An original circuit-level model of two-terminal vanadium dioxide elec-
tron devices exhibiting electronic hysteresis is presented. Such devices
allow realisation of very compact relaxation nano-oscillators that
potentially can be used in bio-inspired neurocomputing. The proposed
model is exploited to determine the parameters, values that ensure
stable periodic oscillations.

Introduction: Vanadium dioxide (VO_2) is a correlated electron material
that exhibits abrupt insulator-to-metal (IMT) and metal-to-insulator
(MIT) phase transitions under the application of a critical electric field
(transitions may also be triggered by other stimuli, such as strain and
thermal excitation) [1]. These phase transitions correspond to a large
and abrupt change in electrical conductivity. Recently, it has been
shown that connecting a VO_2 device with a resistor and a capacitor,
of proper resistance and capacitance values, it is possible to fabricate
very compact relaxation nano-oscillators [2]. The simple fabrication
flow, the easy scalability with the possibility to achieve high packaging
density, make VO_2 nano-oscillators promising candidates to integrate a
large array of coupled oscillators for bio-inspired neurocomputing [2, 3].

However, electrical measurements reveal that the switching-like be-
haviour of a two-terminal VO_2 comes at the expense of a hysteresis
with the IMT and the MIT transition being driven by two different crit-
ical electric field values. In this Letter, we provide, for the first time, a
circuit-level model of the hysteresis mechanism which is a key to pre-
dicting and controlling the dynamics of relaxation oscillators built on
VO_2 devices. We show how the proposed model, after being tuned
with experimental data, allows us to predict the parameter values for
which relaxation oscillation occurs.

Circuit-level model of VO_2 device: The I–V current–voltage character-
istic of a two-terminal VO_2 exhibits a typical hysteretic behaviour [2]
(see Fig. 2). For relatively low voltages, the device has an insulator-like
behaviour with $I = V/R_{ins}$, where R_{ins} is the resistance in the insulator
state. When the electric voltage approaches the critical value V_H, a
IMT occurs and the current I abruptly increases. This transition is
abrupt but not instantaneous with a finite intrinsic electronic switching
time [4]. In the metallic state, we have that $I = V/R_{met}$ with a much
smaller resistance $R_{met} \ll R_{ins}$. In this condition, to bring the device
back to the insulator state, the voltage should be reduced under the
lower value $V_L < V_H$ at which the inverse MIT occurs.

To model this hysteretic behaviour, we adopt the 'driving-point
equivalent' circuit [5] with abrupt (but not instantaneous) transitions
described in Fig. 1. In the model, the 'state' (i.e. insulator or metal) of
the device is decided by a voltage comparator (CMP) and it is stored
into the intrinsic capacitor C_o. The value of the product $\tau_o = R_o \times C_o$ is
selected so as to reproduce the time constant of the phase transition
detected in laboratory measurements. The CMP output voltage V_o,
whose value ranges from 0 to 1 V, is transferred by a controlled
voltage source at the CMP input terminal +, where it selects the value
of V^+ between the two critical levels V_L and V_H. The device current is
given by the current $I = I_f = G_f V_f$ that flows into the feedback resistor
R_f shown in Fig. 1. The electrical conductance of such a resistor is con-
trolled by the state voltage V_c as follows

$$G_f = G_{min}(1 - V_c) + G_{max} V_c \qquad (1)$$

where $G_{min} = 1/R_{ins}$ and $G_{max} = 1/R_{met} \gg G_{min}$ are the device conduct-
ance in the insulator and metal state, respectively. In the insulator
state, the CMP output is $V_o = 1$ V and $V_c = 0$ (at the DC regime). In
this state, $V^+ = V_H$ and $G_f = G_{min}$. Hence, if V is gradually increased
till to approach V_H, it initiates a IMT. After the device has completed
its transition to the metallic state, the CMP output becomes $V_o = 0$ V
and $V_c = 1$. In this state, $G_f = G_{Max}$ and $V^+ = V_L$. Thus, it is only when
V is reduced below $V^+ = V_L$ that a reverse MIT can occur completing
the cycle of the hysteresis characteristic. Fig. 2 shows the I–V character-
istics obtained by simulating the circuit-level model in Fig. 1 for a
2 µm-length VO_2 device. The parameters of the model were extracted
by fitting the experimental curve of the device as follows: $R_{ins} =$
50 kΩ, $R_{met} = 1$ kΩ, $V_L = 0.5$ V, $V_H = 5.9$ V and $\tau_o = 100$ ns. The

device is connected in series to a DC voltage source, sweeping from 0
to 7 V, and to a resistance R_s. Fig. 2 shows the I–V curves for the two
cases: (i) zeros series resistance $R_s = 0$ and (ii) finite series resistance
$R_s = 200$ Ω. Simulations correctly predict the negative-slope shape of
the I–V curve in the presence of a finite resistance R_s [2]. We conclude
that, after being tuned with the experimental data of a given VO_2 device,
the proposed circuit-level model is able to realistically reproduce its
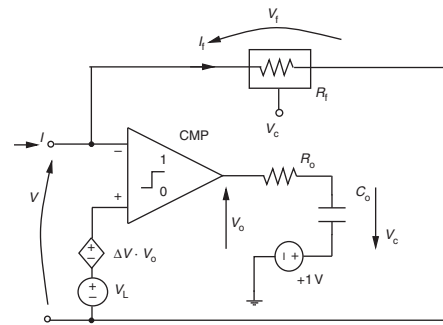electronic behaviour seen at the external terminals.

Fig. 1 *Device model*
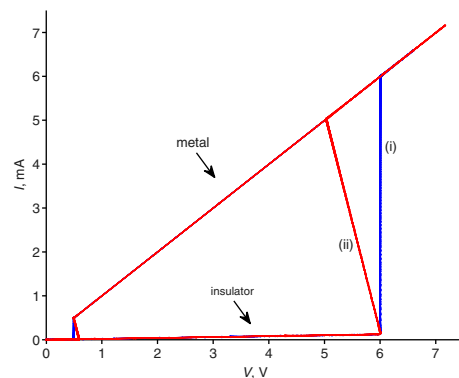$\Delta V = V_H - V_L$ gives width of voltage hysteresis

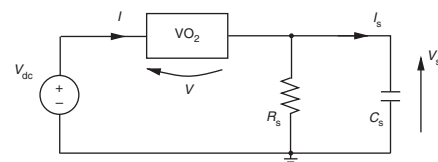**Fig. 2** *Simulated VO_2 characteristics for cases: (i) $R_s = 0$ Ω and (ii) $R_s = 200$ Ω*

Fig. 3 *Circuit of relaxation oscillator*

Relaxation oscillation: The circuit of the relaxation oscillator can be
obtained by connecting the VO_2 device to an external resistor R_s and
capacitor C_s, as shown in Fig. 3. The circuit is supplied by a DC
voltage source V_{dc}. We performed extensive electrical simulations of
the circuit in Fig. 3, with the VO_2 device model in Fig. 1, to investigate
the effect of electrical parameters. Simulations showed that a minimum
supply voltage V_{dc} value is needed to obtain oscillations. Furthermore,
they revealed that, for a given voltage supply, the occurrence of relax-
ation oscillations critically depends on the value of the external series
resistance R_s; oscillations only arise when R_s is chosen within a very
precise interval of values. With the VO_2 device in our example and
with $V_{dc} = 14$ V, stable oscillations occur for 30 kΩ $< R_s <$ 55 kΩ.
Finally, the value of the external capacitance C_s affects the frequency
of the oscillation. This behaviour was fully confirmed by laboratory
measurements. Fig. 4 reports the simulated and the measured $V_s(t)$
waveforms for a relaxation oscillator with electrical parameters $V_{dc} =$
14 V, $C_s = 300$ pF and $R_s = 47$ kΩ (i.e. within the numerically-predicted
oscillation interval); simulations and measurements match with excel-
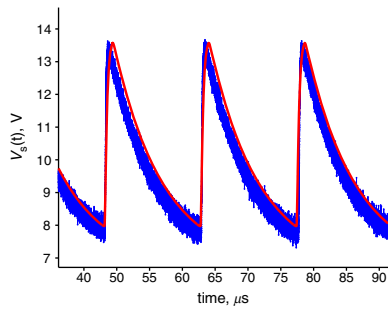lent accuracy and yield a relaxation oscillation at ≈68 kHz.

Fig. 4 *Simulated (solid line) and measured (dotted line) $V_s(t)$ waveforms for $R_s = 47\ k\Omega$*

Conclusion: In this Letter, the electrically induced hysteresis mechanism in VO_2 devices has been modelled for the first time with a driving-point equivalent circuit. The circuit can incorporate the experimentally characterised critical voltages, variable device resistances and switching time of the hysteretic response. We have shown how the proposed model enables realistic electrical simulations of oscillators built on the hysteretic device and provides guidance on how to achieve a robust design. It is believed that the circuit-level model will play a fundamental role in designing large arrays of VO_2 oscillators.

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